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# Anisotropic Mesh Deformation using Stiffness Fields

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## Introduction

To be able to solve fluid-structure interaction problems, the mesh needs to be deformable. One classic method to perform this is the spring analogy. But this algorithm lacks robustness leading to shapes collapsing even for small or medium motions. To overcome these limitations, this work proposes a method based on:

1. Spring Analogy.
2. Stiffness field.
3. Iterative Grid Stretching

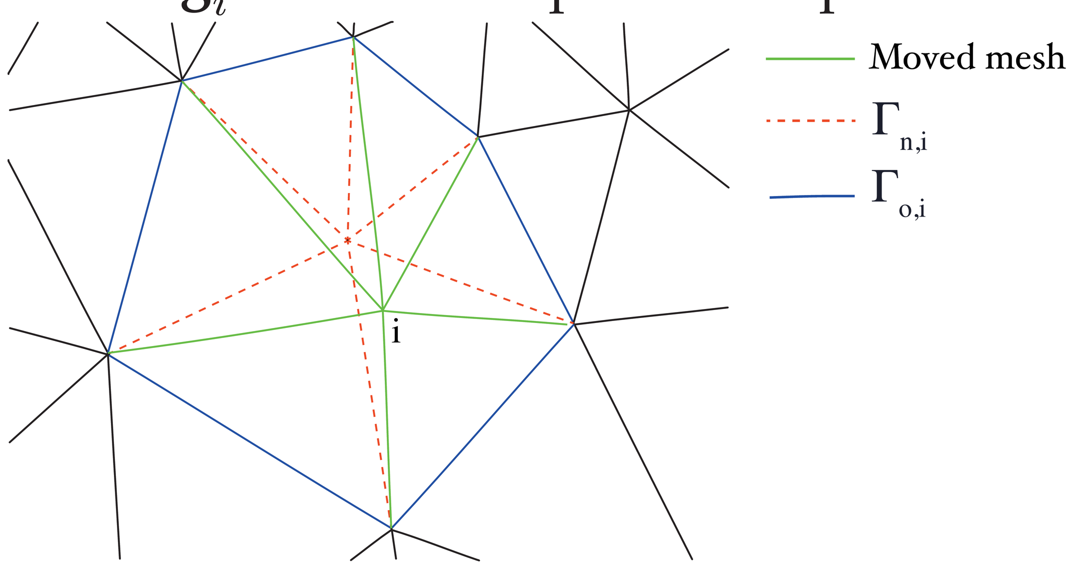
## Spring Analogy Method

This method consists of putting a spring in all faces of all grid elements. A spring network where all point displacements are inter-dependent is created. In this work we consider unstructured, hybrid meshes.

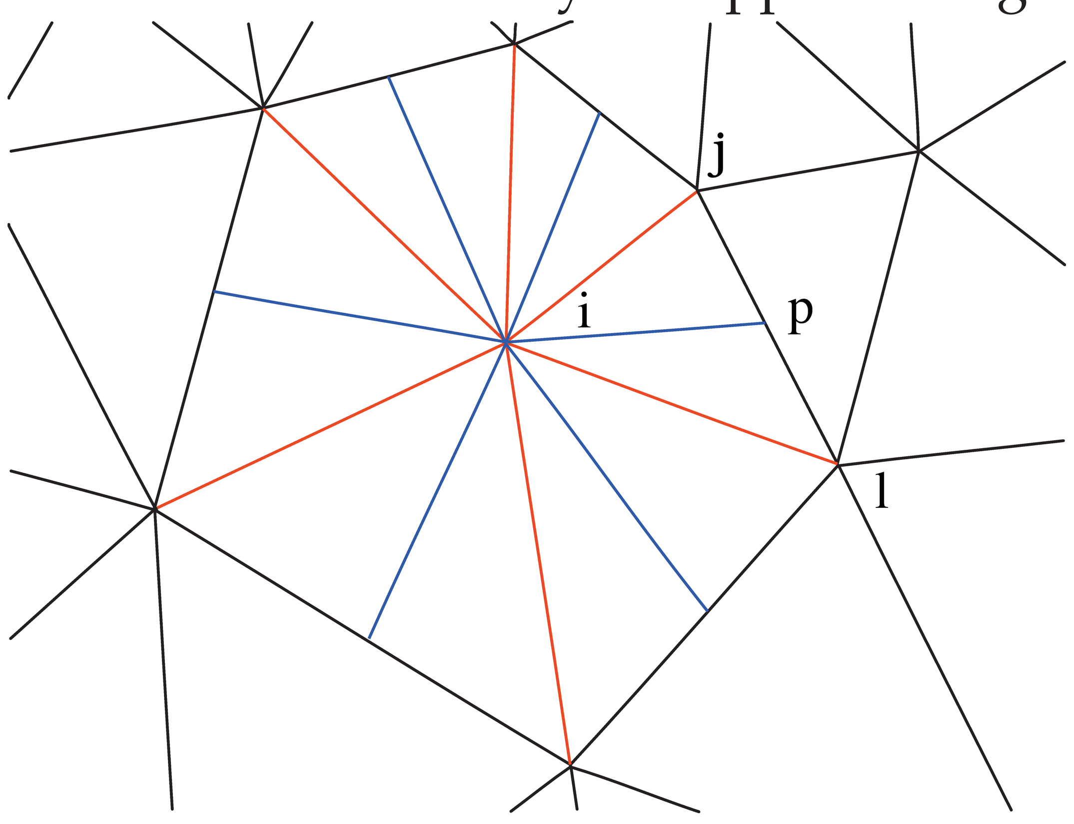
Each point $i$ is described by a two dimensional motion $\mathbf{U}_i = U_{ix} + U_{iy}$. Its connected edges are part of $\Gamma_{n,i}$ and its opposite edges are in $\Gamma_{o,i}$. In $\Gamma_w$ and $\Gamma_{m,w}$ the displacement is imposed by the aerofoil movement by:

$$\mathbf{U}_i = \mathbf{g}_i, \ \forall i \in \{\Gamma_w, \Gamma_{m,w}\} \quad (1)$$

where $\mathbf{g}_i$ is the imposed displacement.




A triangle composed of spring edges is deformable, so the system needs to prevent points from reaching an opposite edge. The ball vertex method[2] adds a virtual spring from each point $i$ to each respective opposite edge centre point $p$. Hence, each point is now bound by its opposite edges.




Each "spring" will have a stiffness of edges given by:

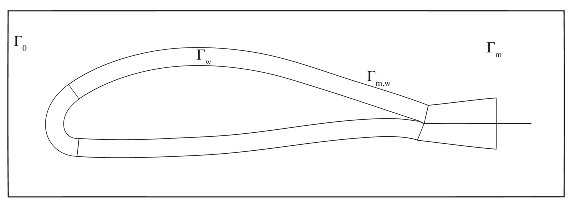
$$k_{i,j} = \frac{1}{L_{i,j}}, \ \forall \{i,j\} \quad (2)$$

$$k_{i,p} = \frac{1}{L_{i,p}}, \ \forall \{i,p\} \quad (3)$$

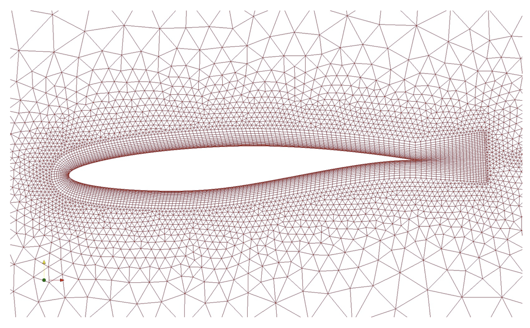
## Stiffness field

We consider an aerofoil mesh defined by the following regions:

- $\Gamma_{m,w}$ : discretized aerofoil wall
- $\Gamma_w$ : discretized quadrilateral zone
- $\Gamma_0$ : discretized far-field
- $\Gamma_m$ : triangular mesh



Mesh Regions



RAE 2822 Grid

The grid shown above, was generated with GMSH[1]. Initial tests of the algorithm show that it works well with translations. But when there rotations are present in mesh regions of many constraints, triangles collapse despite the virtual springs. Therefore, the stiffness field is not suitable for large rotations. To overcome this problem the stiffness is modified by linking it to the distance from the boundary:

$$k_i = \begin{cases} \frac{2}{L_i}\left(\frac{10}{d_i+\beta} - 9\right)\left(100e^{-2d_i^2}+1\right), & i \in \Gamma_{m,w} \\ \frac{1}{L_i}\left(\frac{10}{d_i+\beta} - 9\right)\left(100e^{-2d_i^2}+1\right), & i \in \Gamma_m \\ \frac{1}{L_i}\left(e^{-2d_i^2}+1\right), & d_i \geq 1-\beta \end{cases} \quad (4)$$

## Iter. Grid Stretching

If an edge length keeps increasing, its stiffness will decrease. So, cells stretch more easily if a displacement is decomposed into several sub-steps. So all displacements are decomposed into sub-steps and the stiffness is now given by:

$$\begin{cases} \Delta k_i = k_{i,t_0} - k_{i,t} \\ k_i = k_{i,t_0} + \Delta k_i \end{cases} \quad (5)$$

The mesh displacement at each iteration is driven to a steady state solution. For each vertex $i$ the new equilibrium position is given by:

$$\sum_{j=1}^{n_e} \mathbf{k}_{i,j}\mathbf{U}_j + \sum_{p=1}^{n_o} \mathbf{k}_{i,p}\mathbf{U}_p - \\ - \left(\sum_{j=1}^{n_e} \mathbf{k}_{i,j} + \sum_{p=1}^{n_o} \mathbf{k}_{i,p}\right)\mathbf{U}_i = 0 \quad (6)$$

where $n_o$ is the number of opposite edges. To close the system, an equation to keep a point $p$ in the center of each edge $j, l$ is needed:

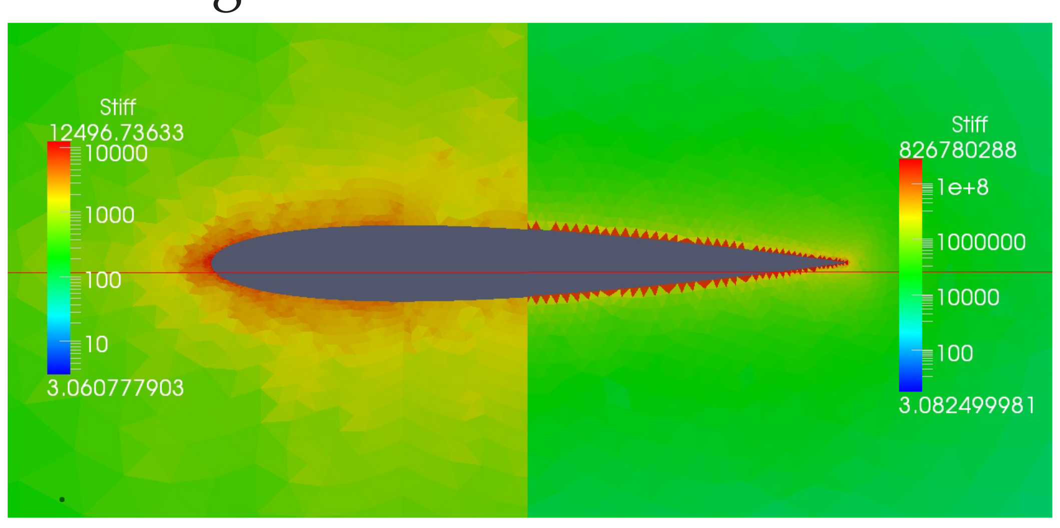
$$\mathbf{U}_p - \frac{\mathbf{U}_j + \mathbf{U}_l}{2} = 0, \ \forall p \in \{\Gamma_{o,i}\} \quad (7)$$

The system of equations is solved using *spsolve* from ScyPy.

## Results

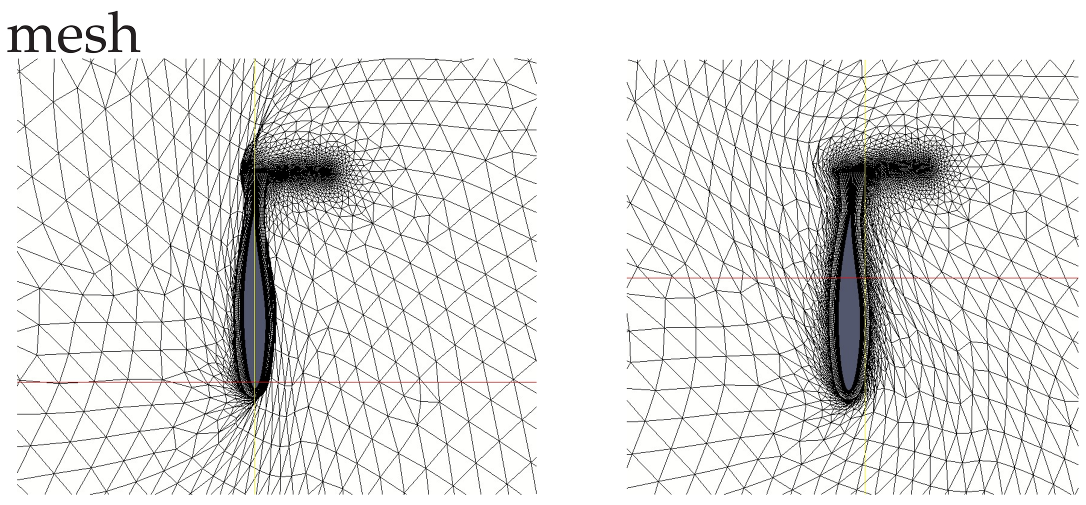
Results for a 45° rotation of the RAE 2822 mesh - impact of new stiffness field, eq.4

Original   New stiffness field

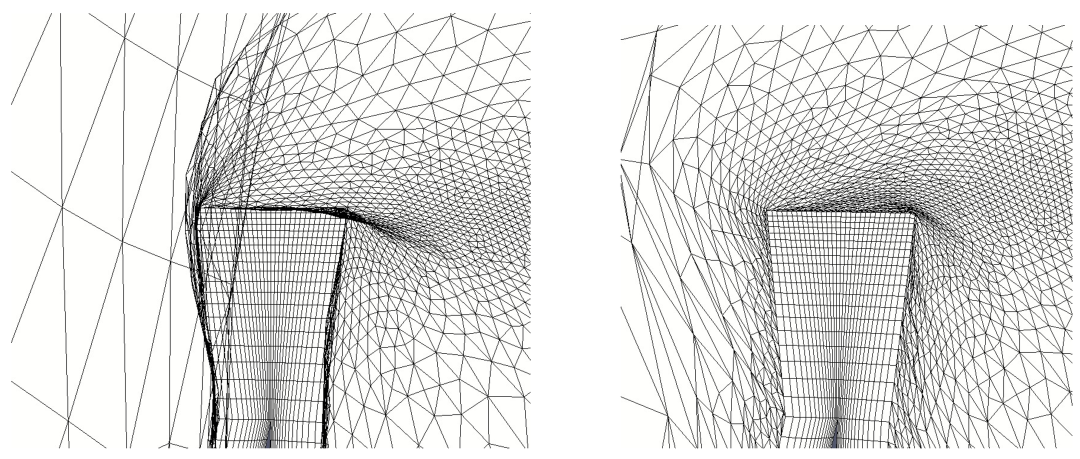



Original   New stiffness field

Results for a 90° rotation of the RAE 2822 mesh



New stiffness field   New field & Stretching



New stiffness field   New field & Stretching

The modified stiffness field allows cells near rigid zones to have increase stiffness, hence improving quality for large rotations. The inclusion of sub-iterations to achieve a final displacement allowed obtaining valid grids for rotations up to 90°.

## Conclusion

The original ball vertex method lacks robustness for large rotations. This work proposes a new way to define the stiffness field. The new field allows rotations for angles up to 50°. For larger displacements, sub-iterations stiffness calculations were required. The spring analogy method is straightforward to implement, hence future work will explore further refinements to this method.

## References

[1] C. Geuzaine and J.-F. Remacle. Gmsh: a three-dimensional finite element mesh generator with built-in pre- and post-processing facilities, *International Journal for Numerical Methods in Engineering*, 79(11), pp. 1309-1331, 2009

[2] C. Bottasso, D. Detomi and R. Serra, The Ball-Vertex Method: a new simple Spring Analogy Method for Unstructured Dynamic Meshes *Comp. Meth. in Applied Mechanics and Engineering*, 194(39-41):4244-4264, 2005